Standard 1

Functions

A2.1.1   Find the zeros, domain and range of a function.

Example: $f(x) = x^3 - 3x^2 - x + 3$. What is the domain and range of this function?

A2.1.2   Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function f below, find the indicated value, if possible.

- where f is represented by the set of ordered pairs {(3, 5), (2, -3), (1, 7), (0, 2)}, find the value of $f(1)$.
- where $f(x) = \sqrt{4-x}$, find the value of $f(2)$ and $f(8)$.

A2.1.3   Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 2

Linear and Absolute Value Equations, Inequalities and Functions

CORE STANDARD
Linear and Absolute Value Equations and Inequalities

Solve systems of linear equations and inequalities in three variables by substitution and elimination. Solve problems that can be modeled using systems of linear equations. Solve equations and inequalities involving absolute value.

[Standard Indicators: A2.2.1, A2.2.2, A2.2.4]

A2.2.1   Solve systems of linear equations and inequalities in three variables by substitution and elimination.

Example: Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$ and $5y - 6z = 9$.

A2.2.2   Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.

Example: Each week you can work no more than 20 total hours between the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

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A2.2.3   Graph piecewise-defined functions.

Example: Graph the function $f(x) = \begin{cases} x + 2 \text{ if } x \leq 0 \\ 3x - 1 \text{ if } x > 0 \end{cases}$.

A2.2.4   Solve equations and inequalities involving the absolute value of a linear function.

Example: Solve the inequality $|x - 5| \geq 8$ and graph the solution.

Standard 3
Quadratic Equations and Functions

CORE STANDARD
Complex Numbers

Add, subtract, multiply and divide complex numbers.

[Standard Indicator: A2.3.1]

CORE STANDARD
Quadratic Equations and Functions

Solving Quadratic Equations
Solve quadratic equations in the complex number system. Solve problems that can be modeled using quadratic equations and functions.

[Standard Indicators: A2.3.2, A2.3.5]

Graphing Quadratic Functions
Graph quadratic functions. Determine how the graph of a parabola changes if *a*, *b* and *c* are changed in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

[Standard Indicators: A2.3.3, A2.3.4]

A2.3.1   Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.

Example: Let $z = 7 - 4i$ and $w = 10 + 6i$. Graph z, w and $z + w$. Prove that the numbers $0, z, w$ and $z + w$ are the vertices of a parallelogram on the complex plane.

A2.3.2   Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

A2.3.3   Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry.

   Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.4   Determine how the graph of a parabola changes if a, b and c change in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.

   Example: Write the equation of the parabola with vertex $(3, 6)$ and y-intercept 2 in vertex form.

A2.3.5   Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.

   Examples:
   - Write the equation of the parabola with vertex $(3, 6)$ and y-intercept 2 in vertex form.
   - Describe similarities and differences in the graphs of $y = 2x^2$ and $y = 2(x - 1)^2 + 3$ without first graphing the equations.

Standard 4

Polynomial Expressions, Equations and Functions

CORE STANDARD
Polynomial Equations and Functions

Solving Polynomial Equations
Solve polynomial equations by factoring. Solve problems that can be modeled using polynomial equations.

[Standard Indicators: A2.4.4, A2.4.6]

Writing Polynomial Equations
Perform arithmetic operations, including long division, on polynomials. Find a polynomial when given its roots and use the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression to solve problems.

[Standard Indicators: A2.4.3, A2.4.7]

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A2.4.1   Analyze, describe and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.

   Example: Determine by inspection the end behavior of the graph of the function.
   $f(x) = -2x^3 + x^2 + 4x - 5$.

A2.4.2   Use the binomial theorem to expand binomial expressions raised to positive integer powers.

   Example: Expand $(x + 2)^4$.

A2.4.3   Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.

       Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.4.4   Factor polynomials completely and solve polynomial equations by factoring.

       Example: Solve $x^3 + 27 = 0$ by factoring.

A2.4.5   Use graphing technology to find approximate solutions for polynomial equations.

       Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.4.6   Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable.

       Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Then use your results to give a formula for the volume of the box.

A2.4.7   Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression.

       Example: Write an equation that has solutions $x = 2$, $x = 5i$, and $x = -5i$.

Standard 5

Rational and Radical Expressions, Equations and Functions

CORE STANDARD
Rational Functions

Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator. Solve problems that can be modeled using equations involving rational functions.

[Standard Indicator: A2.5.2]

A2.5.1   Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

       Example: Find the equations of the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x+1}{x+5}$.

A2.5.2   Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.

       Example: Simplify $\dfrac{x^2 - 4}{x^5} \div \dfrac{x^3 - 8}{x^8}$.

A2.5.3   Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}} \, y^{\frac{-2}{3}} \right)^6$ in simplest form. Assume all variables are positive.

A2.5.4   Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

A2.5.5   Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

A2.5.6   Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable.

Example: Two students working independently can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

Standard 6
Exponential and Logarithmic Functions

CORE STANDARD
Exponential and Logarithmic Equations

Use laws of exponents to derive laws of logarithms. Use laws of logarithms to solve problems. Solve exponential and logarithmic equations. Solve problems that can be modeled using equations involving exponents and logarithms.

[Standard Indicators: A2.6.2, A2.6.3, A2.6.4]

A2.6.1   Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.6.2   Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that $\log(2) = a$ and $\log(3) = b$, find $\log(36)$ in terms of a and b.

A2.6.3   Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

A2.6.4   Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate.

> **Example:** The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million and 400 million. What do you notice about these time periods?

Standard 7
Sequences and Series

CORE STANDARD
Sequences and Series

Find specific terms of arithmetic and geometric sequences. Find partial sums of arithmetic and geometric series. Solve problems that can be modeled using arithmetic and geometric series.

[Standard Indicators: A2.7.1, A2.7.3, A2.7.4]

A2.7.1   Write the recursive formula for arithmetic and geometric sequences and find specific terms of arithmetic and geometric sequences.

> **Example:** Find the tenth term of the arithmetic sequence 3, 7, 11, 15… .

A2.7.2   Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.

> **Example:** Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162… .

A2.7.3   Find partial sums of arithmetic and geometric series.

> **Example:** In the last example (A2.7.2), find the sum of the first 10 terms.

A2.7.4   Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.

> **Example:** A restaurant has square tables that seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Data Analysis and Probability

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CORE STANDARD
Combinatorics and Probability

Use permutations, combinations and other counting methods to determine the number of ways that events can occur. Calculate the probability of compound events and analyze probabilities to interpret odds and risks of events.

[Standard Indicators: A2.8.2, A2.8.4]

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A2.8.1   Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

    Example: Use technology to simulate throwing two dice 500 times. Use the results to estimate the probability of rolling a 7 and then use the diagram of the sample space to find the theoretical probability.

A2.8.2   Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risks of events.

    Example: When a die is rolled three times, what is the probability of obtaining a 6, followed by any even number, followed by a 4?

A2.8.3   Know and apply the characteristics of the normal distribution.

- Identify settings in which the normal distribution may be useful.

- Determine whether a set of data appears to be uniform, skewed or normally distributed.

- Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two or three standard deviations from the mean.

    Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that a randomly selected student's SAT score is greater than 600?

A2.8.4   Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

    Example: There are five students who work in a bookshop. If the bookshop needs three people to operate, how many days straight could the bookstore operate without the same group of students working twice?

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PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof
- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication
- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

• Create and use representations to organize, record and communicate mathematical ideas.

• Select, apply and translate among mathematical representations to solve problems.

• Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

• Know and apply appropriate methods for estimating the results of computations.

• Use estimation to decide whether answers are reasonable.

• Decide when estimation is an appropriate strategy for solving a problem.

• Determine appropriate accuracy and precision of measurement in problem situations.

• Use properties of numbers and operations to perform mental computation.

• Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

• Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.

• Technology can contribute to concept development, simulation, representation, communication and problem solving.

• The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.

– Graphing calculators should be used to enhance middle school and high school students' understanding and skills.

– The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts.* Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.

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